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Sørensen, John Dalsgaard; Stang, Birgitte Friis Dela; Svensson, Staffan

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J. D. Sørensen, Birgitte Dela Stang,
Staffan Svensson

Calibration of load duration factor k_{mod}

John Dalsgaard Sørensen, Aalborg University, Aalborg, Denmark
Birgitte Dela Stang, Danish Building and Urban Research, Hørsholm, Denmark
Staffan Svensson, Lund University, Sweden

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1 Introduction

Load duration effects with respect to long term decrease of strength are very important for structural timber. This report describes how the load duration effect can be determined on basis of simulation of realizations of the time varying load processes. Wind, snow and imposed loads are considered and the stochastic models are formulated in accordance with the load models in the Danish structural codes, DS 409, [3] and DS 410, [4]. Three damage accumulation models are considered, namely Gerhards model, [5], Barret & Foschi's model, [7] and Foschi & Yao's model [10]. The parameters in these models are fitted using data relevant for Danish structural timber, Hoffmeyer, [6].

Load duration factors, k_{mod} are calibrated using a probabilistic method requiring that the lifetime reliability for representative limit states are the same for situations where load duration is taken into account and situations where short term strength models are used.

2 Load models

2.1 Snow load

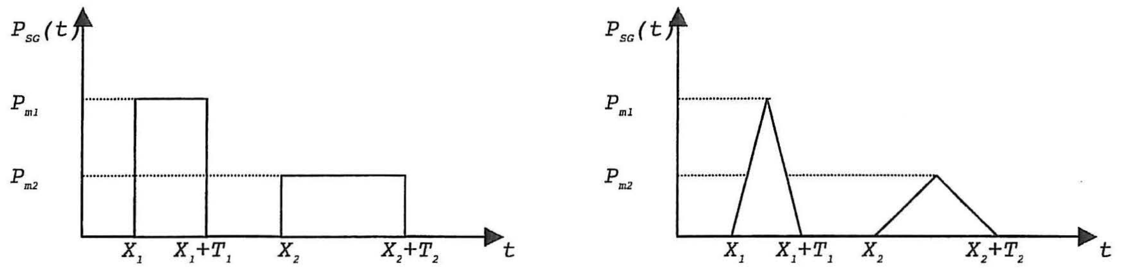


Figure 1. Snow load model. Left: rectangular snow load. Right: triangular snow load.

The annual maximum snow load on a structure is determined from

$$P_S = CP_{SG} \tag{1}$$

where

P_{SG} is the annual maximum snow load on the ground

C is a shape factor

The snow model for the snow load on the ground is illustrated in figure 1. The following assumptions are made:

1. the occurrence of snow packages at times $X_1, X_2, ...$ is modeled by a Poisson process. The duration between snow packages is therefore exponential distributed with expected value $1/\lambda$, where λ is the expected number of snow packages per year.
2. the magnitude of the maximum snow load P_m in one snow package is assumed to be Gumbel distributed with expected value μ_P and standard deviation σ_P.
3. the length of a snow package T is equal to $X_T P_m$, i.e. proportional to the maximum snow load of the snow package. X_T is assumed to be Exponential distributed with expected value μ_{X_T}.
4. the time variation of a snow package is assumed to be rectangular or triangular, see figure 1.

2.2 Wind load

The annual maximum wind pressure on a structure is determined from

$$P_{W,\max} = CP_W \tag{2}$$

where

P_W is the annual maximum wind pressure

C is a shape factor

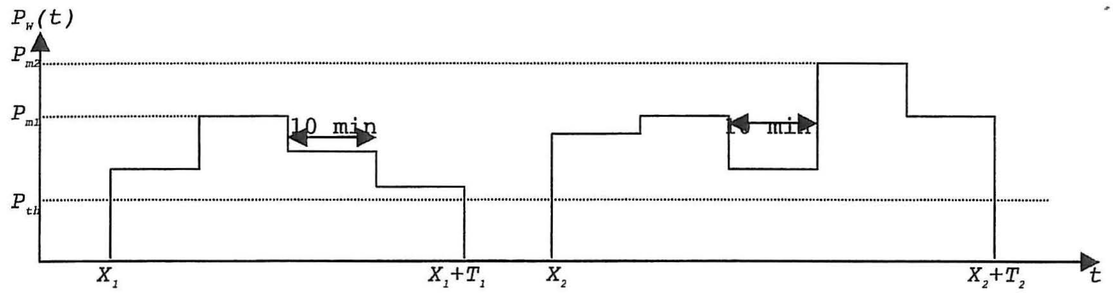


Figure 2. Wind load model. Time variations in each 10-minutes period are not shown.

The wind model is illustrated in figure 2. The following assumptions are made:

1. the occurrence of storms at times $X_1, X_2, ...$ is modeled by a Poisson process. The duration between storms thus becomes exponential distributed with expected value $1/\lambda$, where λ is the expected number of storms per year.
2. the magnitude of the maximum wind pressure P_m in one storm is assumed Gumbel distributed with expected value μ_P and standard deviation σ_P.
3. the length of one storm T (in sequences of 10 minutes periods) is equal to $X_T P_m$. X_T is assumed to be Exponential distributed with expected value μ_{X_T}.
4. the magnitude of the wind pressure P_t in a 10-minutes period in a given storm is modeled as $P_t = P_m - X_P(P_m - P_{th})$ where P_{th} is a lower threshold on wind pressures measured (e.g. proportional to (13 m/s)^2). X_P is assumed to be Beta distributed with expected value μ_{X_P} and standard deviation σ_{X_P}. The wind pressures in one storm are limited to be between a lower threshold measured (e.g. proportional to (average wind velocity = 13 m/s)^2) and the maximum value for the storm, implying that $0 \le X_P \le 1$. It is assumed that the sequence of the 10-minutes periods is unimportant.
5. the time history of the wind pressure $P(t)$ during each 10 minute period is modeled using the wind spectrum and wind action model in DS410 [4].

2.3 Imposed load

Imposed load is modeled in accordance with the JCSS load model, see [8] and CIB W81, [9], and consists of sustained loads and intermittent loads. The following assumptions are made:

1. the sustained load changes at times $X_1, X_2, ...$ are modeled by a Poisson process. The duration between changes is exponential distributed with expected value λ_{sus}.
2. the magnitude of the sustained load P_{sus} is assumed Gamma distributed with expected value μ_{sus} and standard deviation $\sigma_{sus} = \sqrt{\sigma_v^2 + \sigma_{u,sus}^2 \frac{A_0}{A}\kappa}$ with parameters defined in table 1.
3. the intermittent loads occurring at times $X_1, X_2, ...$ are also modeled by a Poisson process. The duration between the intermittent loads is thus exponential distributed with expected value λ_{int}.

4. the magnitude of the intermittent loads P_{int} is assumed Gamma distributed with expected value μ_{int} and standard deviation $\sigma_{\text{int}} = \sqrt{\sigma_{u,\text{int}}^2 \frac{A_0}{A}\kappa}$ with parameters defined in table 1. The time length of the intermittent loads is t_{int}.

		Sustained load				Intermittent load			
	A_0 [m^2]	μ_{sus} [kN/m^2]	σ_v [kN/m^2]	$\sigma_{u,sus}$ [kN/m^2]	λ_{sus} [year]	μ_{int} [kN/m^2]	$\sigma_{u,\text{int}}$ [kN/m^2]	λ_{int} [year]	t_{int} [days]
Office	2	0.5	0.3	0.6	5	0.2	0.4	0.3	1-3
Residence	2	0.3	0.15	0.3	7	0.3	0.4	1.0	1-3

Table 1. Parameters for imposed load, see [8]. $A = 5$ m^2 and $\kappa = 1.778$.

3 Strength model

3.1 Short term strength

The initial (short term) bending strength is assumed to be LogNormal distributed with coefficient of variation equal to 15% or 20%, which are the basic COV's used in Denmark for laminated and structural timber, respectively.

3.2 Damage models

3.2.1 Gerhards model

The strength of wood depends on the duration of the applied load. The following damage model proposed by Gerhards [5] is used:

$$\frac{d\alpha}{dt} = \exp\left(-A + B\frac{\sigma}{R_0}\right) \tag{3}$$

where

α is the accumulated damage. Failure occurs when $\alpha \geq 1$

σ is the applied stress

R_0 is the initial (short term) strength

A, B are constants

The solution of the differential equation (3) assuming constant load σ and setting $\alpha = 1$ give

$$\frac{\sigma}{R_0} = \frac{A}{B} - \frac{\ln 10}{B}\log t = a - b\log t \tag{4}$$

where

$$a = \frac{A}{B} + \varepsilon \qquad b = \frac{\ln 10}{B} \tag{5}$$

and

ε models the model uncertainty. ε is assumed to be Normal distributed with expected value equal to 0 and standard deviation σ_ε

Based on test results from a test program carried out on structural timber (Nordic spruce) in 4 point bending, see Hoffmeyer [6], the parameters in table 1 are estimated using the Maximum Likelihood technique. Simultaneously the standard deviations modeling statistical uncertainties are estimated, see table 2. The corresponding correlation coefficients are:
$\rho(a,b)$=0.90, $\rho(a,\sigma_\varepsilon)$=0.08 and $\rho(b,\sigma_\varepsilon)$=0.02.

Figure 3 shows the data and the fit obtained using the parameters in table 2.

	a	b	σ_ε
Estimate	0.90	0.0495	0.0206
Standard deviation	0.0055	0.0016	0.0021

Table 2. Maximum Likelihood estimates and standard deviation of parameters in Gerhards model.

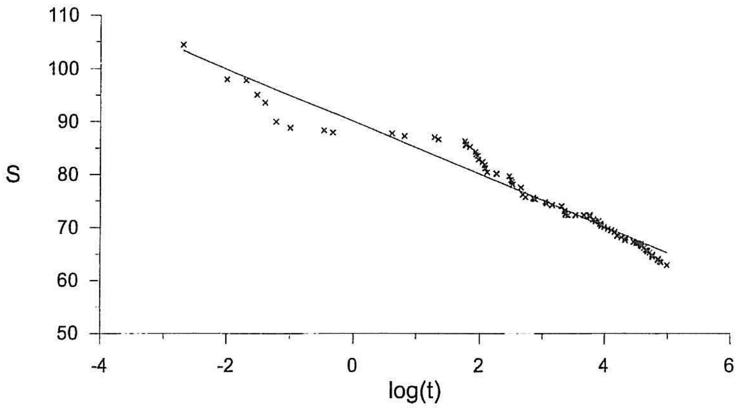


Figure 3. Fit of data to Gerhards model. x-axis: $\log(t)$, t in hours and y-axis: $S = \sigma / R_0$ in %. x: data; full line: fit.

The solution of the differential equation (3) using constant load σ is

$$\alpha = t \exp\left(-A + B \frac{\sigma}{R_0} \right) + C \tag{6}$$

where C is a constant. If R is considered as the residual strength corresponding to the damage α, then using that $\alpha = 0$ and 1 for $\frac{R}{R_0} = 1$ and 0:

$$\frac{R}{R_0} = \frac{1}{B}\left[1 + (1-\alpha)(\exp B - 1)\right] \tag{7}$$

3.2.2 Barret and Foschis model

The damage model suggested by Barret & Foschis [7] is written:

$$\begin{aligned} \frac{d\alpha}{dt} &= A\left(\frac{\sigma}{R_0} - \eta\right)^B + C\alpha \qquad ; \frac{\sigma}{R_0} > \eta \\ \frac{d\alpha}{dt} &= 0 \qquad\qquad\qquad\qquad\quad ; \frac{\sigma}{R_0} \leq \eta \end{aligned} \tag{8}$$

where

A, B, C constants,

□ threshold ratio (typically equal to 0.5)

□ stress

R_0 initial (short term) strength.

Solution of the differential equation (8) using constant load σ and setting $\alpha = 1$ give:

$$\frac{\sigma}{R_0} = \left(\frac{A}{C}(\exp(C \cdot t) - 1) \right)^{-\frac{1}{B}} + \eta = a(\exp(\exp(b) \cdot t) - 1)^c \tag{9}$$

where

$$a = \exp\left(\ln\left(\left(\frac{A}{C}\right)^{-\frac{1}{B}}\right) + \varepsilon\right) \qquad b = \ln C \qquad c = -\frac{1}{B} \qquad (10)$$

and

ε models the model uncertainty. ε is assumed to be Normal distributed with expected value equal to 0 and standard deviation σ_{ε}

Based on test results in Hoffmeyer [6], the parameters in table 3 are estimated using the Maximum Likelihood technique. Simultaneously the standard deviations modeling statistical uncertainties are estimated, see table 3. The corresponding correlation coefficients are:
$\rho(a,b)$=0.82, $\rho(a,c)$=0.65, $\rho(b,c)$=0.64, $\rho(a,\sigma_{\varepsilon})$= -0.09, $\rho(b,\sigma_{\varepsilon})$= -0.16 and $\rho(c,\sigma_{\varepsilon})$= -0.11.

Figure 4 shows the data and the fit obtained using the parameters in table 3.

	a	b	c	σ_{ε}
Estimate	0.221	-9.14	-0.063	0.075
Standard deviation	0.0033	0.079	0.002	0.006

Table 3. Maximum Likelihood estimates and standard deviations of parameters in Barret & Foschi's model.

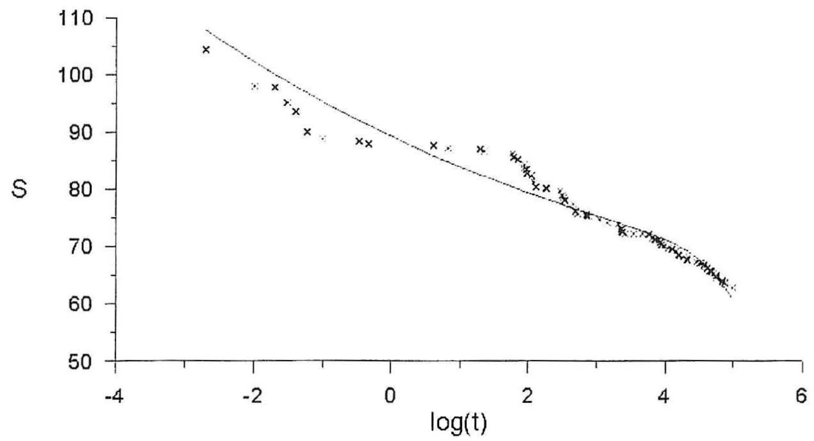


Figure 4. Fit of data to Barret & Foschi's model. x-axis: $\log(t)$, t in hours and y-axis: $S = \sigma / R_0$ in %. x: data; full line: fit.

The solution of the differential equation (8) using constant load σ is

$$\alpha = \frac{A}{C}\left(\frac{\sigma}{R_0} - \eta\right)^B (\exp(Ct) - 1) + D \qquad (11)$$

where D is a constant. If R is considered as the residual strength corresponding to the damage α, then using that $\alpha = 0$ and 1 for $\frac{R}{R_0} = 1$ and η:

$$\frac{R}{R_0} = \eta + \left[(1-\alpha)(1-\eta)^B\right]^{\frac{1}{B}} \qquad (12)$$

3.2.3 Foschi an Yao's model (Canadian model)

The damage model suggested by Foschi & Yao [10] is written:

$$\begin{aligned}
\frac{d\alpha}{dt} &= A\left(\frac{\sigma}{R_0}-\eta\right)^B + C\left(\frac{\sigma}{R_0}-\eta\right)^D \alpha \qquad ; \frac{\sigma}{R_0} > \eta \\
\frac{d\alpha}{dt} &= 0 \qquad\qquad\qquad\qquad\qquad\qquad\quad ; \frac{\sigma}{R_0} \leq \eta
\end{aligned} \tag{13}$$

where

A, B, C, D constants,

□ threshold ratio (typically equal to 0.5)

□ stress

R_0 initial (short term) strength.

Solution of the differential equation with short term ramp load ($\sigma = kt$) until failure with initial strength R_0 gives (assuming rate of loading is large and C small), see Köhler & Svensson [11]

$$A = \frac{k(B+1)}{R_0(1+\eta)^{(B+1)}} \tag{14}$$

The time until failure t_f can then be determined from, see Köhler & Svensson [11]:

$$t_f = \frac{\sigma}{k} + \frac{1}{C\left(\frac{\sigma}{k}-\eta\right)^D}\ln\left(\frac{1+\lambda}{\alpha_0+\lambda}\right) \tag{15}$$

where

$$a_0 = \left(\frac{\frac{\sigma}{R_0}-\eta}{1-\eta}\right)^{B+1} \tag{16}$$

and

$$\lambda = \frac{k(B+1)}{CR_0(1-\eta)^D}\left(\frac{\sigma}{R_0}-\eta\right)^{B-D} \tag{17}$$

Equation (15) can also be written

$$t_f = \ln\left(\frac{\sigma}{k} + \frac{1}{C\left(\frac{\sigma}{k}-\eta\right)^D}\ln\left(\frac{1+\lambda}{\alpha_0+\lambda}\right)\right) + \varepsilon \tag{18}$$

where

ε models the model uncertainty. ε is assumed to be Normal distributed with expected value equal to 0 and standard deviation σ_ε

Based on the test results in Hoffmeyer [6], the parameters in table 4 are estimated using the Maximum Likelihood technique. Simultaneously the standard deviations modeling statistical uncertainties are estimated, see table 4. The corresponding correlation coefficients are:
$\rho(B,C)$=0.38, $\rho(B,D)$=0.12, $\rho(C,D)$=0.98, $\rho(B,\sigma_\varepsilon)$= 0.00, $\rho(C,\sigma_\varepsilon)$= 0.01 and $\rho(D,\sigma_\varepsilon)$= 0.00.

Figure 5 shows the data and the fit obtained using the parameters in table 4.

	B	C	D	σ_ε
Estimate	27.3	9.78	5.44	0.35
Standard deviation	0.92	7.57	0.46	0.03

Table 4. Maximum Likelihood estimates and standard deviations of parameters in Barret & Foschi's model.

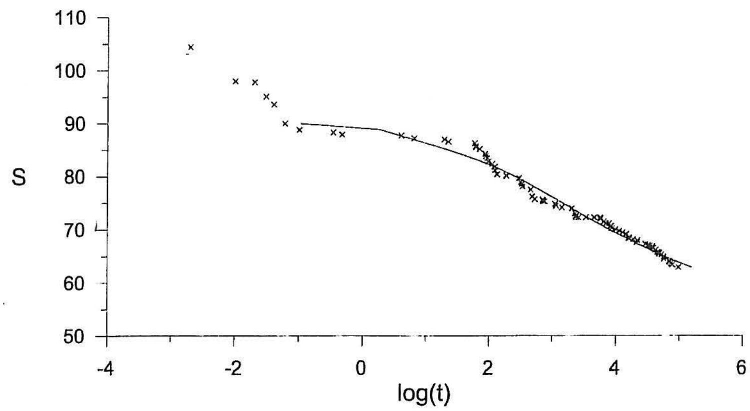


Figure 5. Fit of data to Barret & Foschi's model. x-axis: $\log(t)$, t in hours and y-axis: $S=\sigma / R_0$ in %. x: data; full line: fit.

If R is considered as the residual strength corresponding to the damage α, then:

$$\frac{R}{R_0}=\eta+(1-\eta)(1-\alpha)^{1/(1+B)} \tag{19}$$

4 Calibration of load duration factor

In this section it is described how a load duration factor can be calibrated using a probabilistic approach. Probabilistic methods modeling of the strength by stochastic models have been used in e.g. [5], [7] and [12], but in the following a full probabilistic approach is described where all uncertainties related to strength, model and loads are included in a way consistent with the background for the partial safety factors in the Danish structural codes.

	Distribution	Expected value	Coefficient of variation	Characteristic value
R_0	Lognormal	1	V_R =0.15 V_R =0.20	0.77 0.72
X_R	Lognormal	1	0.05	1
G	Normal	1	0.1	1
Q	Gumbel	1	V_Q=0.2/0.4	$1-V_Q\frac{\sqrt{6}}{\pi}[0.5772+ln\{-ln(0.98)\}]$
Z_R	Lognormal	1	$V_Z = 0.0 / 0.1$	1
A	Normal	Table 1, 2 or 3	Table 1, 2 or 3	
B	Normal	Table 1, 2 or 3	Table 1, 2 or 3	
C	Normal	Table 1, 2 or 3	Table 1, 2 or 3	
ε	Normal	0	Table 1, 2 or 3	

Table 5. Stochastic model.

The following short-term limit state function is considered:

$$g = zR_0X_R - ((1-\kappa)G + \kappa Q) \tag{20}$$

where

z design parameter

κ coefficient, $0 \le \kappa \le 1$

R_0 short term strength

X_R model uncertainty for short term strength

G permanent load

Q variable load

The corresponding design equation is:

$$\frac{zR_k}{\gamma_m} - ((1-\kappa)\gamma_G G_k + \kappa\gamma_Q Q_k) = 0 \tag{21}$$

where

R_k characteristic value for short term strength (5 % quantile)

G_k characteristic value for permanent load (mean value)

Q_k characteristic value for variable load (98 % quantile in one year maximum distribution)

γ_m partial safety factor for material parameter (=1.5/1.64 if coeff. of variation = 0.15/0.20)

γ_G partial safety factor for permanent load (=1.0)

γ_Q partial safety factor for variable load (=1.5 for environmental load)

The design variable z is determined from (21) and next, the reliability index β is calculated on the basis of (20) and the stochastic model in table 5. It is noted that in the Danish codes the reference one-year reliability index is β =4.8 and V_Q=0.2 and 0.4 for imposed and environmental load, respectively.

The following long-term limit state equation is used:

$$g = 1 - Z_R \alpha\left(R_0, G, Q, A, B, C, SR(z), \eta, \kappa, T_L\right) \tag{22}$$

where

Z_R model uncertainty for long term strength with mean 1 and coefficient of variation V_Z

α damage function. Gives the accumulated damage after T_L=50 years with a time varying variable load $Q = Q(t)$

T_L design life time (= 50 years)

A, B, C parameters in damage accumulation model

SR stress ratio $= \dfrac{(1-\kappa)G + \kappa Q}{zR}$

η threshold value

R_0 short term strength

G permanent load

$Q = Q(t)$ variable load as function of time

The design equation corresponding to the limit state function (22) is:

$$\frac{zR_k}{\gamma_m} k_{\text{mod}} - \left((1-\kappa)\gamma_G G_k + \kappa \gamma_Q Q_k\right) = 0 \tag{23}$$

where

k_{mod} load duration duration factor

In order to take into account in the short-term model the decrease with time of the strength due to accumulated damage the following alternative limit state equation is used:

$$g = z\frac{R}{R_0} R_0 X_R - \left((1-\kappa)G + \kappa Q\right) \tag{24}$$

where R is the residual strength corresponding to the damage α at time t. $\dfrac{R}{R_0}$ is determined from (7), (12) or (19). A simple, conservative alternative is to use $(1-\alpha)R_0$ instead of R in (24):

$$g = z(1-\alpha)R_0 X_R - \left((1-\kappa)G + \kappa Q\right) \tag{25}$$

where α is the damage function obtained from (3), (10) or (13). In (25) the strength is reduced linearly by the damage.

The limit state functions (24) and (25) are used with the design equation (23).

The k_{mod} factor is calibrated by the following steps:

1. Calculate the short term reliability index β_{50}^{S} for a 50 year reference period using the limit state function (20) and the design equation (21). β_{50}^{S} is calculated as function of γ_m by simulation (γ_G and γ_Q are fixed). The design parameter z is determined from (21) and a realization of the limit state function (20) is simulated, see sections 4.1, 4.2 and 4.3. The time to failure, $T_{F,S}$ is determined. The probability of failure is estimated by

$$P_{F,S} = \frac{\text{number of realisations where } T_{F,S} \leq T_L}{\text{total number of realisations}} \tag{26}$$

The corresponding reliability index is determined from

$$\beta_{50}^{S} = -\Phi^{-1}\left(P_{F,S}\right) \tag{27}$$

where $\Phi(\ )$ is the standard Normal distribution function.

2. Calculate the long term reliability index β_{50}^{L} for a 50 year reference period using the limit state function (22) and the design equation (23) and $k_{\text{mod}}=1$. β_{50}^{L} is calculated as function of γ_m by simulation (γ_G and γ_Q are fixed). The design parameter z is determined from (23) and a realization of the limit state function (22) is simulated, see sections 4.1, 4.2 and 4.3. The time to failure, $T_{F,L}$ is determined. The probability of failure is estimated by

$$P_{F,L} = \frac{\text{number of realisations where } T_{F,L} \leq T_L}{\text{total number of realisations}} \tag{28}$$

The corresponding reliability index is determined from

$$\beta_{50}^{L} = -\Phi^{-1}\left(P_{F,L}\right) \tag{29}$$

3. Estimate k_{mod} from

$$k_{\text{mod}} = \frac{\gamma_m^S(\beta)}{\gamma_m^L(\beta)} \tag{30}$$

for a reasonable range of values of the reliability index β corresponding to the 50 year reference period. $\gamma_m^S(\beta)$ is the short term partial safety factor as function of β and $\gamma_m^L(\beta)$ is the long term partial safety factor as function of β

In order to evaluate the time-variant reliability the following three supplementary reliability indices are determined:

$$\beta_{50}^{C} = -\Phi^{-1}\left(P_{F,C}\right) \tag{31}$$

where the probability of failure is estimated by

$$P_{F,C} = \frac{\text{number of realisations where } T_{F,S} \leq T_L \text{ or } T_{F,L} \leq T_L}{\text{total number of realisations}} \tag{32}$$

$$\beta_{50}^{R1} = -\Phi^{-1}(P_{F,R1}) \tag{33}$$

where the probability of failure is estimated by

$$P_{F,R1} = \frac{\text{number of realisations where } T_{F,R1} \leq T_L}{\text{total number of realisations}} \tag{34}$$

and the time to failure, $T_{F,R1}$ is determined using the limit state function (24).

$$\beta_{50}^{R2} = -\Phi^{-1}(P_{F,R2}) \tag{35}$$

where the probability of failure is estimated by

$$P_{F,R2} = \frac{\text{number of realisations where } T_{F,R2} \leq T_L}{\text{total number of realisations}} \tag{36}$$

and the time to failure, $T_{F,R2}$ is determined using the limit state function (25).

4.1 Results for snow load

It is assumed that $\kappa = 1$, i.e. there is no permanent load. Based on data from [1] the shape factor C is modeled as Gumbel distributed with expected value $\mu_C = 1$ and standard deviation $\sigma_C = 0.35$.

Based on recorded Danish snow data over 32 years from five locations (Karup, Skrydstrup, Tirstrup, Værløse and Aalborg) the following data have been estimated:

1. $\lambda = 1.175$ snow packages per year
2. P_m in one snow package: Gumbel distributed with $\mu_P = 0.33$ kN/m^2 and $\sigma_P = 0.21$ kN/m^2.
3. duration of snow package factor: X_T: Exponential distributed with $\mu_{X_T} = 75$ days/ (kN/m^2).

Therefore the annual maximum snow load P_{SG} becomes Gumbel distributed with expected value $\mu = 0.36$ kN/m^2 and standard deviation $\sigma = 0.21$ kN/m^2. The 98% quantile in the annual maximum distribution becomes $P_{SG,0.98} = 0.99$ kN/m^2.

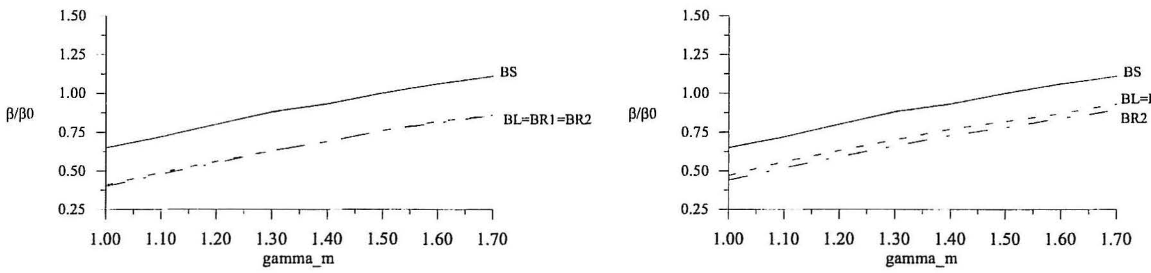
The reliability indices (short and long term) are estimated by simulation. One realization during the design life time (50 years) is simulated as follows:

1. simulate a realization of the model uncertainty C (incl. uncertainty on shape factors).
2. simulate durations between snow packs.
3. simulate magnitudes of maximum snow loads on ground P_m in each snow package.
4. simulate lengths of snow packages T.
5. calculate the time history of the snow load as $Q(t) = C \cdot P(t)$
6. apply the snow load to a damage accumulation law. Calculate:

- the first time where the load is larger than the short term strength (limit state function (20) $\leq$ 0) : $T_{F,S}$
- the first time where the accumulated damage is larger than 1 (limit state function (22) $\leq$ 0) : $T_{F,L}$
- the first time where the load is larger than the damage reduced short term strength (limit state function (24) $\leq$ 0) : $T_{F,R1}$
- the first time where the load is larger than the damage reduced short term strength (limit state function (25) $\leq$ 0) : $T_{F,R2}$

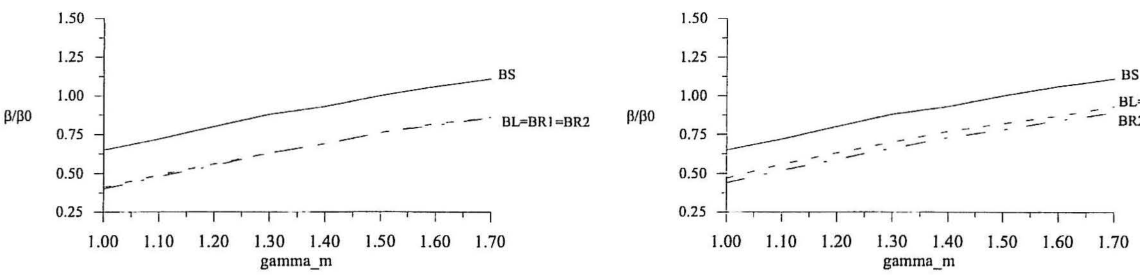
Figure 6 and 7 show the relative reliability indices $\beta(\gamma_m)/\beta_0$ as function of γ_m for rectangular and triangular snow packages and for the Gerhard and the Barret & Foschi damage models. V_R=0.15, V_Z=0 and no statistical uncertainty is included. $\beta_0 = \beta(\gamma_m = 1.5)$ is the reliability index for the short term limit state with $\gamma_m = 1.5$. $\beta_{50}^C = \beta_{50}^L$ for all γ_m values, and is not shown. It is seen that

- the long term reliability is smaller than the short term reliability.
- $\beta_{50}^{R1} \approx \beta_{50}^L$ indicating that damage reduced strength does not change the reliability compared to the long term reliability.
- The conservative model in (25) gives reliability indices β_{50}^{R2} equal to (rectangular snow packages) and slightly less (triangular snow packages) than the reliability indices β_{50}^{R1} based on the damage reduced strength model in (24).
- Gerhards and Barret & Foschi's damage models give almost the same relative reliability levels.



rectangular triangular

Figure 6. Relative reliability index $\beta / \beta 0$ with $\beta 0 = \beta(\gamma_m = 1.5)$ as function of γ_m. BS = β_{50}^S; BL = β_{50}^L; BR1 = β_{50}^{R1}; BR2 = β_{50}^{R2}. Damage model: Gerhard.



rectangular triangular

Figure 7. Relative reliability index $\beta / \beta 0$ with $\beta 0 = \beta(\gamma_m = 1.5)$ as function of γ_m. BS = β_{50}^S; BL = β_{50}^L; BR1 = β_{50}^{R1}; BR2 = β_{50}^{R2}. Damage model: Barret & Foschi.

Figure 8 shows the probability of failure as function of time for Barret & Foschi's damage model, triangular snow packages, V_R=0.20, V_Z=0.1, γ_m=1.3 and no statistical uncertainty. It is seen that

- The probability of failure is slightly nonlinear as function of time, indicating that the annual probability of failure decreases slightly with time.
- The long term probability of failure (based on accumulated damage in (22)) is equal to the probability of failure based on the damage reduced strength in (24), whereas the short term probability of failure (based on the initial strength in (20)) is smaller.

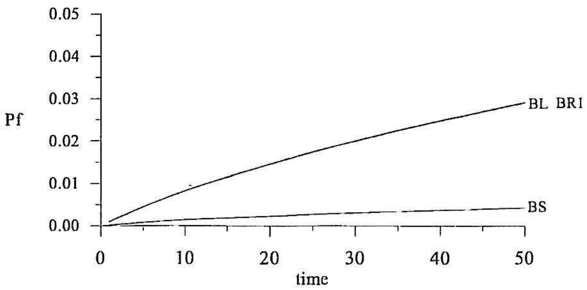


Figure 8. Probability of failure as function of time in years. BS, BL and BR1: limit state functions (20), (22) and (24).

		without stat. unc.		with stat. unc.	
		Rectangular	Triangular	Rectangular	Triangular
Gerhards	V_R=0.15	0.75 / 0.75	0.81 / 0.81	0.75 / 0.75	0.80 / 0.79
	V_R=0.20	0.75 / 0.75	0.81 / 0.82	0.75 / 0.75	0.79 / 0.79
Barret & Foschi	V_R=0.15	0.75 / 0.75	0.81 / 0.81	0.75 / 0.75	0.80 / 0.79
	V_R=0.20	0.75 / 0.75	0.81 / 0.82	0.75 / 0.75	0.79 / 0.79

Table 6. k_{mod} factors for Danish snow load. a / b : a is for V_Z=0.0 and b is for V_Z=0.1.

		without stat. unc.		with stat. unc.	
		Rectangular	Triangular	Rectangular	Triangular
Gerhards	V_R=0.20	0.75	0.81	0.75	0.79
Barret & Foschi	V_R=0.20	0.75	0.81	0.75	0.79
Foschi & Yao	V_R=0.20	0.75	0.87	0.75	0.86

Table 7. k_{mod} factors for Danish snow load. V_Z=0.0.

k_{mod} factors are calculated using (30). Table 6 and 7 shows the calculated k_{mod} factors. It is seen that:

- triangular snow packages give k_{mod}=0.80 – 0.87 and rectangular snow packages give k_{mod}=0.75
- the choice of damage model is not important for the Gerhard and the Barret & Foschi models whereas higher values are observed for the Foschi & Yao model for triangular load

- statistical uncertainty is not important
- the uncertainty of the short term timber strength is not important
- the model uncertainty is not important

4.1.1 Snow load - effect of duration of snow packages

In this section is investigated the effect of using a reduced duration of the snow packages. Table 8 shows the calculated k_{mod} factors for different expected durations of the snow packages. Rectangular snow packages, no statistical uncertainty, V_Z=0.0, Gerhards damage model and V_R=0.15 are used. It is seen that the duration should be decreased to less than 25 % before a small effect is observed.

μ_{X_T} / 75 days/ (kn/m^2)	k_{mod}
1	0.75
0.5	0.75
0.25	0.76
0.20	0.77
0.15	0.78
0.10	0.81
0.05	0.82

Table 8. k_{mod} factors for different expected durations of snow packages.

4.2 Results for wind load

It is assumed that $\kappa = 1$, i.e. there is no permanent load. Based on data from [2] and [4] the shape factor C is modeled as Gumbel distributed with expected value $\mu_C = 1$ and standard deviation $\sigma_C = 0.215$. Based on recorded Danish wind data over 16 years (Sprogø) the following data have been estimated:

1. $\lambda = 9.4$ storms per year.
2. P_m in one storm: Gumbel distributed with coefficient of variation = 0.44.
3. duration of storm factor X_T: Exponential distributed with $\mu_{X_T} = 1.76$ [10 min./MPa].
4. magnitude of the wind pressure factor X_P: Beta distributed with $\mu_{X_P} = 0.58$ and $\sigma_{X_P} = 0.28$.

The annual maximum wind load P_{SG} becomes Gumbel distributed with coefficient of variation = 0.25.

The reliability indices (short and long term) are estimated by simulation. One realization during the design life time (50 years) is simulated as follows:

1. simulate a realization of the model uncertainty C (incl. uncertainty on shape factors).
2. simulate durations between storms.
3. simulate magnitudes of maximum wind pressures P_m in each storm.
4. simulate lengths of storms T.
5. simulate magnitudes of P_t in all 10-minutes periods in storm.
6. simulate the time history of the wind pressure $P(t)$ during each 10 minute period using a 2 sec. discretization and the wind spectrum in DS 410 [4].
7. calculate the time history of the wind load as $Q(t) = C \cdot P(t)$
8. apply the wind load to a damage accumulation law. Calculate:
   - the first time where the load is larger than the short term strength (limit state function (20) $\leq$ 0) : $T_{F,S}$
   - the first time where the accumulated damage is larger than 1 (limit state function (22) $\leq$ 0) : $T_{F,L}$
   - the first time where the load is larger than the damage reduced short term strength (limit state function (24) $\leq$ 0) : $T_{F,R}$

It is assumed that the structure considered has a height of 10 m and is placed in terrain class II. Further it is assumed that no dynamic effects influence the wind load.

Figure 9 shows the relative reliability indices $\beta(\gamma_m)/\beta_0$ as function of γ_m for the Gerhards and the Barret & Foschi damage models. V_R=0.15, V_Z=0 and no statistical uncertainty is included. $\beta_0 = \beta(\gamma_m = 1.5)$ is the reliability index for the short term limit state with $\gamma_m = 1.5$. $\beta_{50}^C = \beta_{50}^L$ for all γ_m values, and is not shown. It is seen that

- The long term reliability is larger than the short term reliability.

- For Gerhards damage model β_{50}^{R1} is slightly smaller than β_{50}^{S} and for Barret & Foschi's damage model $\beta_{50}^{R1} \approx \beta_{50}^{S}$. This indicates a time variant reliability effect such that k_{mod} should not be larger than 1.
- The conservative model in (25) gives reliability indices β_{50}^{R2} less than the reliability indices β_{50}^{R1} based on the damage reduced strength model in (24).
- Gerhards and Barret & Foschi's damage models give almost the same relative reliability levels.

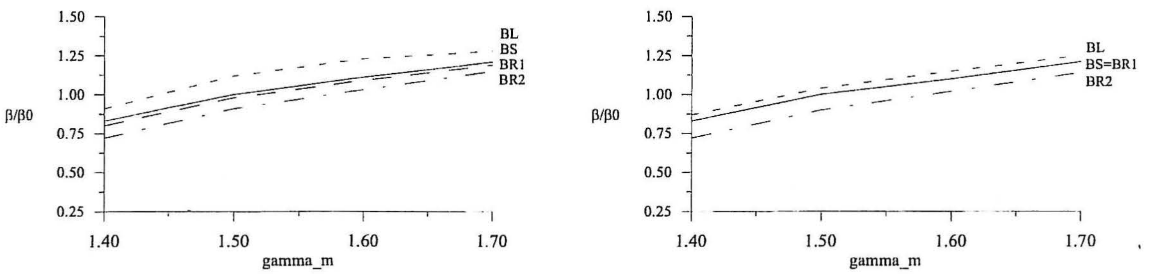


Figure 9. Relative reliability index $\beta / \beta 0$ with $\beta 0 = \beta(\gamma_m = 1.5)$ as function of γ_m. BS = β_{50}^{S}; BL = β_{50}^{L}; BR1 = β_{50}^{R1}; BR2 = β_{50}^{R2}. Damage models: Gerhards and Barret & Foschi.

Figure 10 shows the probability of failure as function of time for Gerhards damage model, V_R=0.20, V_Z=0.1, γ_m=1.4 and statistical uncertainty. It is seen that

- The probability of failure is slightly nonlinear as function of time, indicating that the annual probability of failure decreases with time.
- The long term probability of failure (based on accumulated damage in (22)) is smaller than short term probability of failure (based on the initial strength in (20)) which again is smaller than the probability of failure based on the damage reduced strength in (24).

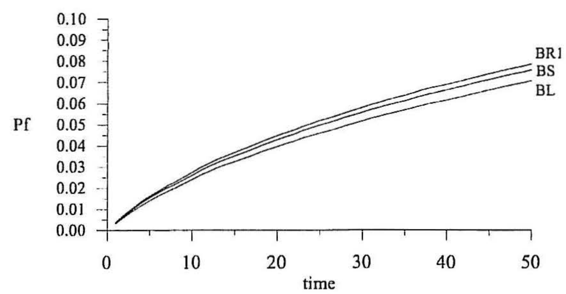


Figure 10. Probability of failure as function of time in years. BS, BL and BR1: limit state functions (20), (22) and (24).

k_{mod} factors are calculated using (30). Tables 9 and 10 show the calculated k_{mod} factors. It is seen that for the k_{mod} factor:

- k_{mod} is approximately 1.05 (from 1.01 to 1.07)
- The Barret & Foschi and Foschi & Yao damage models give slightly larger k_{mod} factors than the Gerhard model

- statistical uncertainty has some importance, especially for the Barret & Foschi damage model
- the uncertainty of the short term timber strength is not important
- the model uncertainty is not important

		without stat. unc.		with stat. unc.	
		V_Z=0.0	V_Z=0.1	V_Z=0.0	V_Z=0.1
Gerhards	V_R=0.15	1.03	1.03	1.01	1.02
	V_R=0.20	1.03	1.03	1.01	1.02
Barret & Foschi	V_R=0.15	1.05	1.07	1.02	1.03
	V_R=0.20	1.04	1.06	1.02	1.03

Table 9. k_{mod} factors for Danish wind load.

		without stat. unc.		with stat. unc.	
		V_Z=0.0		V_Z=0.0	
Gerhards	V_R=0.20	1.03		1.01	
Barret & Foschi	V_R=0.20	1.04		1.02	
Foschi & Yao	V_R=0.20	1.04		1.04	

Table 10. k_{mod} factors for Danish wind load.

4.3 Wind and snow load – combination load

The following load model for wind load is used when snow (or imposed) load is the dominating (extreme) load. The companion wind load applied is therefore modeled on the basis of the instantaneous (daily) wind load.

The instantaneous average 10 minutes pressure on a structure is determined from

$$P_W = CP_{W,I} = C\,0.5\,\rho_a\,V_{10}^2 \tag{37}$$

where

C shape factor

$P_{W,I}$ instantaneous average 10 minutes wind pressure

ρ_a = 1.25 kg/m^3: air density

V_{10} instantaneous average 10 minutes wind velocity

The following assumptions are made:

1. the magnitude of the instantaneous (daily) average 10 minutes velocity V_{10} is assumed Weibull distributed.
2. the time history of the wind pressure $P(t)$ during each 10 minute period is modeled using the wind spectrum in DS410 [4].

Based on recorded Danish wind data the instantaneous (daily) average 10 minutes velocity V_{10} in height = 10 m is modeled as Weibull distributed with expected value = 5.9 m/s and standard deviation = 3.3 m/s, see European Wind Atlas [10].

The load consists of extreme snow load and companion wind load. The corresponding design equation is:

$$\frac{zR_k}{\gamma_m} - \left((1-\kappa)\gamma_G G_k + \kappa\left[\gamma_Q Q_{S,k} + \psi Q_{W,k}\right]\right) = 0 \tag{38}$$

where

$Q_{S,k}$ characteristic value for snow load (98 % quantile in one year maximum distribution)

$Q_{W,k}$ characteristic value for wind load (98 % quantile in one year maximum distribution)

ψ load combination factor for wind load (=0.5)

Load duration factors k_{mod} are calculated using (30) for Barret & Foschi's damage model, triangular and rectangular snow packages, V_R=0.15, V_Z=0, κ =0 and no statistical uncertainty. The result is

k_{mod}=0.91 – 0.94 for constant snow package

k_{mod}=0.94 – 0.97 for triangular snow package

It is seen that the load duration factor is larger than those for extreme snow load alone (0.75 for constant and 0.80 for triangular snow packages), but a little smaller than the load duration factor for wind alone (1.0). It seems to be a reasonable approximation to use the k_{mod} value of the fastest varying load when snow and wind loads are combined.

4.4 Results for imposed load

It is assumed that $\kappa=1$, i.e. there is no permanent load. Based on the data in table 1 the annual maximum imposed load has the characteristics shown in table 11.

	Expected value 50 year max [kN/m^2]	Standard deviation 50 year max [kN/m^2]	COV 50 year max	98% quantile 1 year max [kN/m^2]
Office	3.05	0.88	0.29	2.97
Residence	2.71	0.95	0.35	2.10

Table 11. Parameters for imposed load.

The reliability indices (short and long term) are estimated by simulation. One realization during the design life time (50 years) is simulated as follows:

1. simulate durations between changes in sustained loads and occurrence of intermittent loads.
2. simulate magnitudes of sustained and intermittent loads.
3. calculate the time history of the imposed load snow by summation of the sustained and intermittent loads.
4. apply the imposed load to a damage accumulation law. Calculate:
   - the first time where the load is larger than the short term strength (limit state function (20) ≤ 0) : $T_{F,S}$
   - the first time where the accumulated damage is larger than 1 (limit state function (22) ≤ 0) : $T_{F,L}$
   - the first time where the load is larger than the damage reduced short term strength (limit state function (24) ≤ 0) : $T_{F,R1}$
   - the first time where the load is larger than the damage reduced short term strength (limit state function (25) ≤ 0) : $T_{F,R2}$

Figure 11 and 12 show the relative reliability indices $\beta(\gamma_m)/\beta_0$ as function of γ_m for office and residence loads and for the Gerhards and the Barret & Foschi damage models. V_R=0.15, V_Z=0 and no statistical uncertainty is included. $\beta_0=\beta(\gamma_m=1.3)$ is the reliability index for the short term limit state with $\gamma_m=1.3$. $\beta_{50}^C=\beta_{50}^L$ for all γ_m values, and is not shown. It is seen that

- the long term reliability is smaller than the short term reliability.
- $\beta_{50}^{R1}\approx\beta_{50}^L$ indicating that damage reduced strength does not change the reliability compared to the long term reliability and therefore the load duration factor k_{mod} does not change.
- The conservative model in (25) gives reliability indices β_{50}^{R2} slightly less than the reliability indices β_{50}^{R1} based on the damage reduced strength model in (24).
- Gerhards and Barret & Foschi's damage models give almost the same relative reliability levels.

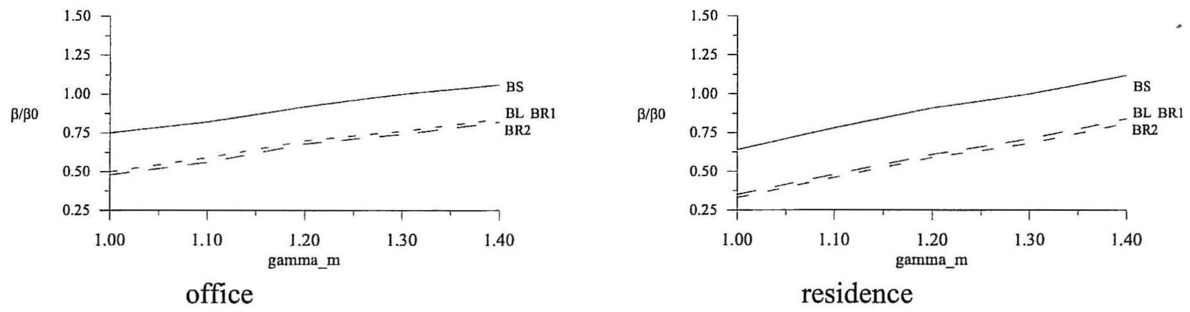


Figure 11. Relative reliability index $\beta / \beta 0$ with $\beta 0 = \beta(\gamma_m = 1.3)$ as function of γ_m. BS = β_{50}^S; BL = β_{50}^L; BR1 = β_{50}^{R1}; BR2 = β_{50}^{R2}. Damage model: Gerhards.

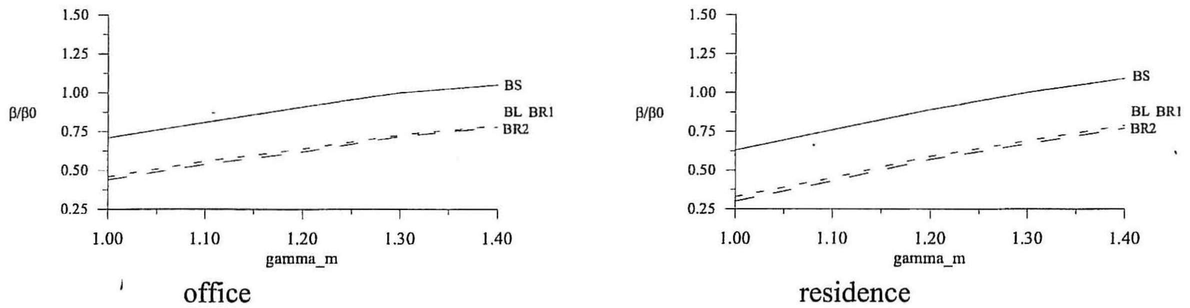


Figure 12. Relative reliability index $\beta / \beta 0$ with $\beta 0 = \beta(\gamma_m = 1.3)$ as function of γ_m. BS = β_{50}^S; BL = β_{50}^L; BR1 = β_{50}^{R1}; BR2 = β_{50}^{R2}. Damage model: Barret & Foschi.

Figure 13 shows the probability of failure as function of time for Gerhards damage model, office loads, V_R=0.15, V_Z=0, γ_m=1.1 and no statistical uncertainty. It is seen that

- The probability of failure is almost linear as function of time, indicating that the annual probability of failure is rectangular with time.
- The short term probability of failure (based on the initial strength in (20)) is much smaller than the probability of failure based on the damage reduced strength in (24) and the long term probability of failure (based on accumulated damage in (22)).

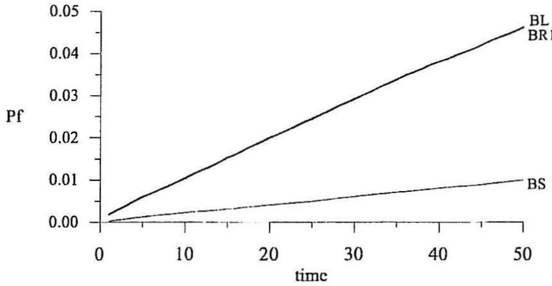


Figure 13. Probability of failure as function of time in years. BS, BL and BR1: limit state functions (20), (22) and (24).

k_{mod} factors are calculated using (30). Tables 12 and 13 show the calculated k_{mod} factors. It is seen that:

- Gerhards and Foschi & Yao's damage model give $k_{\mathrm{mod}} \approx 0.80$ whereas the Foschi damage model gives k_{mod}=0.75-0.80
- k_{mod} is almost the same for office and residence loads
- statistical uncertainty is not important
- the uncertainty of the short term timber strength is not important
- the model uncertainty is not important

		without stat. unc.		with stat. unc.	
		Office	Residence	Office	Residence
Gerhards	V_R=0.15	0.79 / 0.81	0.81 / 0.81	0.79 / 0.79	0.81 / 0.79
	V_R=0.20	0.80 / 0.80	0.79 / 0.80	0.81 / 0.80	0.80 / 0.80
Barret & Foschi	V_R=0.15	0.77 / 0.77	0.80 / 0.80	0.78 / 0.75	0.79 / 0.80
	V_R=0.20	0.76 / 0.78	0.80 / 0.80	0.77 / 0.78	0.80 / 0.80

Table 12. k_{mod} factors for imposed load.

		without stat. unc.		with stat. unc.	
		Office	Residence	Office	Residence
Gerhards	V_R=0.20		0.79		0.80
Barret & Foschi	V_R=0.20		0.80		0.80
Foschi & Yao	V_R=0.20		0.81		0.81

Table 13. k_{mod} factors for imposed load.

5 Summary

It is shown how the load duration effect can be determined on basis of simulation of realizations of the time varying load processes. Stochastic models are presented for wind, snow and imposed loads in accordance with the load models in the Danish structural codes.

Three damage accumulation models are considered, namely Gerhards model, Barret & Foschi's model and Foschi & Yao's model. The parameters in these models are fitted by the Maximum Likelihood Method using data relevant for Danish structural timber and the statistical uncertainty is quantified.

The reliability is evaluated using representative short and long term limit states, and the load duration factor k_{mod} is estimated using the probabilistic model such that equivalent reliability levels are obtained using short and long term design equations. The results are:

- Snow: k_{mod}=0.75-0.80
- Wind: k_{mod}=1.00-1.05
- Imposed: k_{mod}=0.80

If wind and snow loads are combined with snow load as the extreme load then the simulation results indicate that it is reasonable to use the k_{mod} value of the fastest varying load.

Time variant reliability aspect is considered using a simple, representative limit state with time variant strength and simulation of the whole life time load processes. The results indicate that inclusion of the time-variant aspects is unimportant for snow and imposed load, but has some importance for wind load implying that k_{mod}=1.00. The results are based on tests with constant load. However, the loads from the considered variable loads (snow, wind and imposed) have a time-varying behavior. In order to obtain more realistic load duration factors, test should be performed with time-varying load corresponding to typical variations of the real loads. Especially for wind loads with fast changes in the load level an influence on the load duration factor can be expected.

6 Acknowledgements

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